# PART 1: QUESTIONS

**Name:**_____ **Age:**_____ **Id:**_____ **Course:**_____

**Algebra I - Exam 1** **Lesson: 1-3**

**Instructions:**

- Please begin by printing your Name, your Age, your Student Id , and your Course Name in the box above and in the box on the solution sheet.
- You have 90 minutes (class period) for this exam.
- You can not use any calculator, computer, cellphone, or other assistance device on this exam. However, you can set our flag to ask permission to consult your own one two-sided-sheet notes at any point during the exam (You can write concepts, formulas, properties, and procedures, but questions and their solutions from books or previous exams are not allowed in your notes).
- Each multiple-choice question is worth 5 points and each extra essay-question is worth from 0 to 5 points. (Even a simple related formula can worth some points).
- Set up your flag if you have a question.
- Relax and use strategies to improve your performance.

**Exam Strategies to get the best performance:**

- Spend 5 minutes reading your exam. Use this time to classify each Question in (E) Easy, (M) Medium, and (D) Difficult.
- Be confident by solving the easy questions first then the medium questions.
- Be sure to check each solution. In average, you only need 30 seconds to test it. (Use good sense).
- Don't waste too much time on a question even if you know how to solve it. Instead, skip the question and put a circle around the problem number to work on it later. In average, the easy and medium questions take up half of the exam time.
- Solving the all of the easy and medium question will already guarantee a minimum grade. Now, you are much more confident and motivated to solve the difficult or skipped questions.
- Be patient and try not to leave the exam early. Use the remaining time to double check your solutions.

1. Given:

I. $a^2 + b^2 = (a - b)^2 + 2ab$

II. $a^2 - b^2 = (a + b)(a - b)$

III. $a^3 + b^3 = (a + b)^3$

IV. $(a - b)^3 = a^3 - 3a^2b - 3ab^2 + b^3$

a) Only equation IV is false.
b) Only equation III is false.
c) Only equations I and II are true.
d) All equations are false.
e) All equations are true.

Solution: c

I. True

$a^2 - 2ab + b^2 = (a - b)^2$
$a^2 + b^2 = (a - b)^2 + 2ab$

II. True

$(a + b)(a - b) = a^2 - ab + ab - b^2$
$(a + b)(a - b) = a^2 - b^2$

III. False

$a^3 + 3a^2b + 3ab^2 + b^3 = (a + b)^3$
$a^3 + b^3 = (a + b)^3 - 3a^2b - 3ab^2$

IV. False

$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

Thus, only equations I and II are true.

2. Factor $2x^3 - 2y^3$

a) $2(x + y)(x^2 + xy + y^2)$
b) $(x + y)(x^2 - xy + y^3)$
c) $(x + y)(x^2 + 2xy + y^3)$
d) $2(x - y)(x^2 + xy + y^3)$
e) None of above.

Solution: d

$x^3 - 3x^2y + 3xy^2 - y^3 = (x - y)^3$
$x^3 - y^3 = (x - y)^3 + 3x^2y - 3xy^2$
$x^3 - y^3 = (x - y)^3 + 3xy(x - y)$
$x^3 - y^3 = (x - y)[(x - y)^2 + 3xy]$
$x^3 - y^3 = (x - y)[x^2 - 2xy + y^2 + 3xy]$
$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$
Then $2x^3 - 2y^3 = 2(x^3 + y^3)$
$2x^3 - 2y^3 = 2(x - y)(x^2 + xy + y^2)$

Thus, d is the correct answer.

3. Factor $x^2 + 10x + 25$

a) $(5x - 1)(5x + 1)$
b) $(5x + 1)^2$
c) $(x - 5)^2$
d) $(x + 5)^2$
e) $(5x - 2)^2$

Solution: d

$x^2 + 10x + 25$
$\sqrt{x^2}$ $\sqrt{25}$
$x$ $5$
$2(x)(5)$

Then $x^2 + 10x + 25 = (x + 5)^2$

Thus, the correct answer is d.

4. Given

$x^2 + 4xy + 4y^2 = (Ax + By)^2$ then

a) $A + B = 4$
b) $A + B = 3$
c) $A + B = 2$
d) $A + B = 1$
e) None of the Above

Solution: d

$x^2 + 4xy + 4y^2$
$\sqrt{x^2}$ $\sqrt{4y^2}$
$x$ $2y$
$2(x)(2y)$

$x^2 + 4xy + 4y^2 = (x + 2y)^2$

$A = 1$ and $B = 2$

Then $A + B = 3$

Thus, the correct answer is b.

5. Given that $x + y = a,\ x - y = b$ ; where $ab \neq 0$.

$$z = \frac{x^2 - y^2}{x^2 + 2xy + y^2} - \frac{x^2 - y^2}{x^2 - 2xy + y^2} \quad \text{then:}$$

a) $z = \frac{a}{b} - \frac{b}{a}$ b) $z = \frac{a^2 b^2}{a + b}$ c) $z = \frac{a^2 b^2}{a - b}$

d) $z = \frac{b}{a} - \frac{a}{b}$ e) $z = \frac{a^2 + b^2}{ab}$)

Solution: d

$$z = \frac{x^2 - y^2}{x^2 + 2xy + y^2} - \frac{x^2 - y^2}{x^2 - 2xy + y^2}$$

$$z = \frac{(x + y)(x - y)}{(x + y)^2} - \frac{(x + y)(x - y)}{(x - y)^2}$$

$$z = \frac{x - y}{x + y} - \frac{x + y}{x - y} \quad \Rightarrow \quad z = \frac{b}{a} - \frac{a}{b}$$

Thus, the correct answer is d.

6. Factor $a^2 + ab + b^2,\ ab \geq 0$.

a) $(a + b + \sqrt{ab})(a - b + \sqrt{ab})$
b) $(a - b + \sqrt{ab})(a - b + \sqrt{ab})$
c) $(a + b + \sqrt{ab})(a + b + \sqrt{ab})$
d) $(a - b + \sqrt{ab})(a + b - \sqrt{ab})$
e) None of the above

Solution: e

$a^2 + ab + b^2$
$a^2 + 2ab + b^2 - ab$
$(a + b)^2 - (\sqrt{ab})^2 \ ;\ ab \geq 0$
$(a + b + \sqrt{ab})(a + b - \sqrt{ab})$

Thus, the correct answer is e.

7. Simplify $\frac{x^2 - 9}{x^2 - 6x + 9}$

a) $\frac{x + 3}{x - 3}$ b) $\frac{x - 3}{x + 3}$ c) $\frac{x + 3}{x - 2}$ d) $\frac{x - 3}{x + 2}$ e) 1

Solution: a

$$\frac{x^2 - 9}{x^2 - 6x + 9} = \frac{(x + 3)(x - 3)}{(x - 3)^2}$$

$$\frac{x^2 - 9}{x^2 - 6x + 9} = \frac{x + 3}{x - 3}$$

Thus, the correct answer is a.

8. Simplify $\frac{2^{-3}2^4}{2^{-4}2^6}$

a) 0 b) $\frac{1}{2}$ c) 1 d) 4 e) 8

Solution: b

(1) $b^m b^n = b^{m+n}\ ;\ b > 0$

(2) $\frac{b^m}{b^n} = b^{m-n}\ ;\ b > 0$

(3) $\frac{1}{b^{-m}} = b^m\ ;\ b > 0$

$$\frac{2^{-3}2^4}{2^{-4}2^6} \overset{(1).}{=} \frac{2^{-3+4}}{2^{-4+6}} \overset{(2)}{=} \frac{2}{2^2} = 2^{1-2} = 2^{-1} \overset{(3)}{=} \frac{1}{2}$$

Thus, the correct answer is b.

9. The yobibyte is a multiple of the unit byte for digital information. If one yobibyte is $2^{80}$ bytes then:

I. Half of one yobibyte is $2^{80-1}$ bytes.

II. Half of one yobibyte is $\frac{2^{80}}{2}$ bytes.

III. Half of one yobibyte is $2^{79}$ bytes.

IV. Half of one yobibyte is $(2^{80} - 2^{79})$ bytes.

a) I, II, III, and IV are true.
b) I, II, and III are true.
c) Only I and II are true.
d) Only III is true.
e) Only II is true.

Solution: a

(1) $\frac{b^m}{b^n} = b^{m-n}$ ; $b > 0$

I. True

$\frac{2^{80}}{2} = 2^{79}$

II. True (trivial)

III. True

$\frac{2^{80}}{2} \overset{(1)}{=} 2^{80-1} = 2^{79}$

IV. True

$2^{80} - 2^{79} = 2(2^{79}) - 2^{79} = 2^{79}(2-1) = 2^{79}$

Thus, I, II, III and IV are true.

10. Calculate $(3^2 - 3^3)^0$

a) $-18$ b) $-(3^2)$ c) $(3)^0$ d) $3^2$ e) -1

Solution: c

By definition, $b^0 = 1$ ; $b \in \mathbb{R}$.

$(3^2 - 3^3)^0 = 1 = 3^0$

Thus, the correct answer is b.

11. Simplify $x^2 - y^2 - 2x + 10y - 24$

a) $(x + y + 6)(x - y + 4)$
b) $(x + y - 6)(x - y + 6)$
c) $(x + y - 6)(x - y - 4)$
d) $(x + y + 6)(x - y - 4)$
e) None of above.

Solution: e

$x^2 - 2x = x^2 - 2x + 1 - 1 = (x-1)^2 - 1$

$-y^2 + 10y = -(y^2 - 10y) = -(y^2 - 10y + 25 - 25) = -(y-5)^2 + 25$

Then $x^2 - y^2 - 2x + 10y - 24$

$= x^2 - 2x - y^2 + 10y - 24$

$= (x-1)^2 - 1 - (y-5)^2 + 25 - 24$

$= (x-1)^2 - (y-5)^2$

$= (x + y - 6)(x - y + 4)$

Thus, the correct answer is e.

12. Simplify $\frac{3(3^{n+2}) + 2(3^n)}{3^{n+1} - 3^n}$

a) $\frac{29}{2}$ b) $3^{-n}$ c) 13 d) 26 e) $\frac{8}{7}$

Solution: a

(1) $b^m b^n = b^{m+n}$ ; $b > 0$

$\frac{3(3^{n+2}) + 2(3^n)}{3^{n+1} - 3^n} \overset{(1)}{=} \frac{3.3^2.3^n + 2.3^n}{3.3^n - 3^n} = \frac{3^n(27+2)}{3^n(3-1)} = \frac{29}{2}$

Thus, the correct answer is a.

13. Calculate $(x^4)^{(x^4)^{(x^4)}} = 1$ Two possible solutions are:

a) -3 and 3 b) -2 and 2 c) 0 and -1 d) -1 and 2 e) -1 and 1.

Solution: e

$(x^4)^{(x^4)^{(x^4)}} = 1$

$(x^4)^1 = 1$

$x^4 = 1$

$x^4 - 1 = 0$

$(x^2)^2 - 1^2 = 0$

$(x^2 + 1)(x^2 - 1) = 0$

$x^2 + 1 = 0$ or $x^2 - 1 = 0$

$\nexists\, x \in \mathbb{R}$ $x = 1$ or $x = -1$

Thus, the correct answer is e.

14. Calculate $\dfrac{\sqrt{121} - \sqrt{225}}{\sqrt{4}}$

a) $\frac{1}{4}$ b) $-\frac{1}{4}$ c) $\frac{1}{9}$ d) $-\frac{1}{9}$ e) -2

Solution: e

$$\frac{\sqrt{121} - \sqrt{225}}{\sqrt{4}} = \frac{11 - 15}{2} = \frac{-4}{2} = -2$$

Thus, the correct answer is e.

15. Calculate $\sqrt[4]{-27}$ ; Where U = $\mathbb{R}$.

a) There is no solution b) $\frac{1}{3}$ c) 3 d) $-3$ e) $-\frac{1}{3}$.

Solution: a

$\sqrt[4]{-27}$ => Radical = -27
Index = 4 (even)

A negative radical under an even index doesn't belong to $\mathbb{R}$.

$\sqrt[4]{-27} \notin \mathbb{R}$ (No solution)

Thus, there is no solution.

16. Simplify $\sqrt{125}$

a) $2\sqrt{5}$ b) $3\sqrt{5}$ c) $4\sqrt{5}$ d) $5\sqrt{5}$ e) $6\sqrt{5}$

Solution: d

(1) $\sqrt[n]{b^n} = b \;;\; n \geq 0$
(2) $\sqrt[n]{a\,b} = \sqrt[n]{a}\,.\,\sqrt[n]{b}$

$$\sqrt{125} \overset{(2)}{=} \sqrt{5^2 5} \overset{(1)}{=} \sqrt{5^2}\sqrt{5} = 5\sqrt{5}$$

Thus, the correct answer is d.

17. Calculate $\sqrt{a^2\sqrt{a^3}}$ ; Where $a \geq 0$.

a) $a\sqrt[2]{a}$ b) $\sqrt[2]{a^5}$ c) $a^{\frac{7}{4}}$ d) $\dfrac{1}{a^{-\frac{5}{4}}}$ e) All are correct.

Solution: c

(1) $a\sqrt[n]{b} = \sqrt[n]{a^n * b}\;;\; n \geq 0$
(2) $\sqrt[m]{\sqrt[n]{b}} = \sqrt[m*n]{b} \;;\; m \geq 0, n \geq 0$
(3) $\sqrt[n]{b^m} = b^{\frac{m}{n}} \;;\; m \geq 0, n \geq 0$

$$\sqrt{a^2\sqrt{a^3}} \overset{(1)}{=} \sqrt{\sqrt{a^4\,.\,a^3}} \overset{(2)}{=} \sqrt[4]{a^7} \overset{(3)}{=} a^{\frac{7}{4}}$$

Thus, the correct answer is c.

18. Factoring: $-x^2 - (a - b)x + a\,b$

a) $-(x + a)(x + b)$
b) $(a - x)(x - b)$
c) $(a - x)(x - b)$
d) $(a + x)(x - b)$
e) none of the above.

Solution: c

$$\begin{aligned} -x^2 - (a - b)x + a\,b &= -\,[x^2 + (a - b)x - a\,b] \\ &= -\,[x^2 + a\,x - b\,x - a\,b] \\ &= -\,[x(x + a) - b(x + a)] \\ &= -\,(x + a)(x - b) \\ &= (a - x)(x - b) \end{aligned}$$

Thus, the correct answer is c.

19. Calculate $\sqrt{\frac{25}{81}}$

a) $\frac{4}{9}$ b) $\frac{5}{3}$ c) 1 d) $\frac{2}{3}$ e) None of the above.

Solution: e

(1) $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$ ; $n \geq 0$

$$\sqrt{\frac{25}{81}} \overset{(1)}{=} \frac{\sqrt{25}}{\sqrt{81}} = \frac{5}{9}$$

Thus, the correct answer is e.

20. Rationalize $\frac{-2}{1-\sqrt{3}}$

a) $1+\sqrt{3}$ b) $1-\sqrt{3}$ c) $2+\sqrt{3}$ d) $2-\sqrt{3}$ e) 1

Solution: a

$$\frac{-2}{1-\sqrt{3}} = \frac{-2(1+\sqrt{3})}{(1-\sqrt{3})(1+\sqrt{3})} = \frac{-2(1+\sqrt{3})}{1-3} = 1+\sqrt{3}$$

Thus, the correct answer is a.

# PART 2: SOLUTIONS

Consulting

**Name:**_____ **Age:**_____ **Id:**_____ **Course:**_____

## Multiple-Choice Answers

| Questions | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | | | | | |
| 2 | | | | | |
| 3 | | | | | |
| 4 | | | | | |
| 5 | | | | | |
| 6 | | | | | |
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| 8 | | | | | |
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| 17 | | | | | |
| 18 | | | | | |
| 19 | | | | | |
| 20 | | | | | |

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| | Points | Max |
|---|---|---|
| Multiple Choice | | 100 |
| Extra Points | | 25 |
| Consulting | | 10 |
| Age Points | | 25 |
| Total Performance | | 160 |
| Grade | | A |

## Extra Questions

21. Simplify $\dfrac{(x^2-16)(x^2-4x+16)}{x^3+64}$

Solution: $x-4$

$$\frac{(x^2-16)(x^2-4x+16)}{x^3+64} = \frac{(x+4)(x-4)(x^2-4x+16)}{(x+4)(x^2-4x+16)}$$
$$= x-4$$

22. Simplify $\dfrac{x^2-4x+3}{x-1}$

Solution: $x-3$

$$\frac{x^2-4x+3}{x-1} = \frac{x^2-x-3x+3}{(x-1)}$$
$$= \frac{x(x-1)-3(x-1)}{(x-1)}$$
$$= \frac{(x-1)(x-3)}{(x-1)} = x-3$$

23. Rationalize $\dfrac{2}{\sqrt[5]{2}}$

Solution: $\sqrt[5]{16}$

$$\frac{2}{\sqrt[5]{2}} = \frac{2\sqrt[5]{2^4}}{\sqrt[5]{2}\sqrt[5]{2^4}} = \frac{2\sqrt[5]{2^4}}{\sqrt[5]{2^5}} = \frac{2\sqrt[5]{2^4}}{2} = \sqrt[5]{2^4} = \sqrt[5]{16}$$

24. Given $x = 3^{2^3}$ and $y = (3^2)^3$. Calculate $\dfrac{x}{y}$.

Solution: 9

$$x = 3^{2^3} = 3^8$$

$$y = (3^2)^3 = 3^{2*3} = 3^6$$

$$\frac{x}{y} = \frac{3^8}{3^6} = 3^{8-6} = 3^2 = 9$$

25. Given $x = a + \frac{1}{x}$ where $x > 0$. Calculate $x^2 + x^{-2}$

.

Solution: $a^2 + 2$

$x = a + \frac{1}{x}$

$x - \frac{1}{x} = a$

$x - x^{-1} = a$

$(x - x^{-1})^2 = a^2$

$x^2 - 2x(x^{-1}) + (x^{-1})^2 = a^2$

$x^2 - 2 + x^{-2} = a^2$

$x^2 - x^{-2} = a^2 + 2$